Chapter 1 Review of Basic Mathematics

Basic Terms:
Fig 1: Number line
Even Numbers: multiples of 2. Example: $-2, 2, 4, 6 \ldots .248 \ldots$
Odd Numbers: all integers that are not even. Example: $-7 \ldots -3 \ldots 1,3,5 \ldots 273$
Rational Numbers: any number in $\frac{d}{e}$ form where d and e are integers, and $e \neq 0$, note: $-2 = \frac{-2}{1}$, therefore -2 is a rational number.
Irrational Number: Any decimal number with infinite decimal points that do not repeat. Example: $\pi = 3.1415 \ldots \ldots$
Real Number: Any number that can be shown on a number line, including all the numbers discussed above.

Exponents:
w × w × w × w...... 100 times can be written as : w^{100} where w is called the base and 100 is the exponent or the n times.
Rules for solving exponents:

1. $w^0 = 1$
2. $w^a \times w^b = w^{a+b}$
3. $\frac{w^a}{w^b} = w^{a-b}$
4. $\frac{1}{w^a} = w^{-a}$
5. $(w^a)^b = w^{ab}$
6. $\left(\frac{w}{z}\right)^a = \frac{w^a}{z^a}$

Practical application:
Compound Interest: $A = P\left[1 \pm \frac{r}{100}\right]^t$ where A = Total Amount, P = Initial amount, r = rate of change/interest rate (% per year), t = Time in years, + when the rate in increasing, - when the rate in decreasing.
Example: Williams gets €500 prize, and he deposits this amount in his ABN AMRO bank account which pays 6% p.a. interest. How much will he get after 5 years?
⇨ $A = 500\left[1 + \frac{6}{100}\right]^5 = 500[1.06]^5 \cong €669.11$

Basic Algebra Reminders:

1. $(-a).b = a.(-b) = -a.b$

2. $(-a).(-b) = a.b$

3. $a(b+c) = a.b + a.c$

4. $a.a^{-1} = 1, a \neq 0$
5. $\frac{a}{s} \pm \frac{b}{t} = \frac{at \pm bs}{st}$
6. $(a+b)^2 = a^2 + 2ab + b^2$
7. $(a-b)^2 = a^2 - 2ab + b^2$
8. $(a+b)(a-b) = a^2 - b^2$
9. $(a)^{W/z} = \sqrt[z]{a^W}$

Application of Algebra Reminders:
Factoring: $\frac{9x^2-4}{3x-2} = \frac{(3x+2)(3x-2)}{(3x-2)}$ = 3x+2
Fractions: $\frac{6d}{4} + \frac{3}{2} - 2d = \frac{6d+3\times2-2d\times4}{4} = \frac{6-2d}{4} = \frac{3-d}{2}$
Fractional Power: $\frac{\sqrt{48}}{\sqrt{3}} = \frac{\sqrt{4\times4\times3}}{\sqrt{3}} = \frac{4\sqrt{3}}{\sqrt{3}} = 4$

Inequalities:
1. $p > s \text{ and } q < 0, \text{then } pq < sq$

2. When two sides of an inequality are multiplied by a negative number, the direction of the inequality is reversed. Example: 30b > 20c, when the inequality is multiplied by a negative number -2, the inequality becomes -60b < -40c

Example: $\frac{6a-2}{-6} \leq 10$

$=> \textit{multiplying both sides by} -5, \frac{6a-2}{-5} \times -5 \geq 10 \times -5,$

$=> 6a - 2 \geq -50,$

$=> 6a \geq -50 + 2$

$=> a \geq \frac{-48}{6} => a \geq -8$

3. Sign Diagram: Sign diagram is used to show all the possible values for an inequality.

Using the example, sign diagram will be explained:

$\frac{(a-6)}{a-3} > 2 - a$

$\frac{(a-6)}{a-3} - 2 + a > 0$ Take the common denominator and then solve the numerator,

$\frac{a^2-4a}{a-3} > 0$,

$\frac{a(a-4)}{a-3} > 0$ Now this equation one can use the sign diagram

To sign of variable 'a' will determine the sign diagram for a, (a-4) & (a-3). Now, when a = 0, the inequality is not true, thus a≠0 (denoted by • in the sign diagram). When a<4, then a-4 is negative, when a>4 a-4 is positive. When a<3, then a-3 is negative, when a>3 a-3 is positive. Now, we multiply the signs in the diagram to give the final sign diagram of the complete equation. Ex: when a = 2, a-4 is negative, a-3 is negative, so, $\frac{a(a-4)}{a-3}$ is positive.

Fig.2:

Double Inequalities: Remember that all things should be done to both sides of the equality.
$-2 < 4a + 2 < 18$,

⇨ $-2 - 2 < 4a < 18 - 2$

⇨ $\frac{-4}{4} < a < \frac{16}{4}$

⇨ $-1 < a < 4$

Intervals:

There are four different types of intervals. An interval is a set of numbers that lies between two points on a line.

There are two main types of intervals:

1) Open intervals: ex. 2 < x <4
   In this case, the interval consists of all x's greater than 2 and smaller than 4. Note that x does not equal 2 or 4! The notation of this type of interval is (2,4).

2) Closed intervals: ex. 2 ≤ x ≤ 4
   In this case, the interval consists of all x's greater than or equal to 2 and smaller than or equal to 4. The notation of this type of interval is: [2,4].

However, an interval does not have to be completely closed or open. An interval can have one endpoint included and one endpoint excluded- these are called half-open intervals.
Ex. 1: 2 ≤ x ≤ 4.
In this case, x is greater than or equal to 2 (closed) and smaller than 4 (open).The notation of this interval would be [2, 4). These intervals are often shown on a number line; where the endpoints of a closed interval are dots and the endpoints of an open interval are the ends of the arrows:
Fig.3:
There is one more type of interval- the unbounded interval. The example of an unbounded inequality you will see most is that of infinity: [2, ∞)
This simply says: x is greater than or equal to 2. There is no upper limit of x- x can continue on till infinity.

|a|

Because distance can never be a negative number, we call the distance between 0 and 6 the absolute value of 6. The absolute value can be written as $|a|$, and if there is a negative number inside the brackets, for example $|-2|$, it simply equals 2.

We also encounter the absolute value sign when solving inequalities- here are two examples: $|4-2x| \leq 8$. First, we know that this can be written as: $-8 \leq 4-2x \leq 8$, due to the properties of the absolute value. After this step you simply subtract 4 on both sides, and divide by negative 2 on both sides (reminder: if you divide or multiply by a negative number the direction of the inequality signs changes) and the final answer will be: $6 \geq x \geq -2$, in other words, $-2 \leq x \leq 6$.