

$$x = Se^{pn}, \text{ where } S \text{ is a constant}$$

2. $\dot{x} + px = q$ for all values of n ,

$$x = Se^{-pn} + \frac{q}{p}, \text{ where } S \text{ is a constant}$$

3. $\dot{x} + px = qx^2$ for all values of n ,

$$x = \frac{p}{q - Se^{pn}} \text{ where } S \text{ is a constant}$$

Chapter 10 Rate of Interest and Values

Repayment of Mortgage

Let us look at a sample question which would sound like, A person has taken a mortgage for € A that he will pay over t installments at the rate of $r\%$ compounded annually. How much will he pay per installment?

From the previous section, Let p be the amount for each installment so

$$\frac{p}{r} \times 100 \left[1 - \frac{1}{\left(1 + \frac{r}{100}\right)^n} \right] = A$$

Example: Let us use the example of the next door neighbor who has taken a mortgage of € 100000 that he will pay over 4 installments at the rate of 20% compounded annually. How much will he pay per installment?

$$\frac{p}{20} \times 100 \left[1 - \frac{1}{\left(1 + \frac{20}{100}\right)^4} \right] = 100000$$

$$p \times 5 \left[1 - \frac{1}{(1.20)^4} \right] = 100000$$

$$p = \frac{100000}{5} \times 0.5177$$

$$p = 10354$$

Now to find the amount per installment or p we can just use:

$$p = \frac{rA}{1 - (1 + r)^{-n}}$$

To find the number of periods required to pay back the loan at given amount per installment we can use:

$$n = \frac{\ln p - \ln (p - rA)}{\ln (1 + r)}$$

Understanding Internal Rate of Return:

Just remember the following formula where Initial investment is A , and the returns per period is p_1, p_2, \dots, p_n for n periods, The rate of return be r

$$A = \frac{p_1}{(1 + r)^1} + \frac{p_2}{(1 + r)^2} + \dots + \frac{p_n}{(1 + r)^n}$$

To make the calculation easier assum $(1 + r)^{-1} = x$, therefore rewrite the formula

$$A = p_1x + p_2x^2 + \dots + p_nx^n$$

Then first solve for x and substitute $(1+r)^{-1} = x$, to find the value of r

Chapter 11 Multi Variables and their Functions

Understanding Surfaces and Distance

For a 3 dimensional figure, the general equation for a plane in space is given by:

$$px + qy + rz = s$$

Now let us see its economical use,

We can consider each plane or axis to represent a product, then the space is called budget plane, where p, q and r are cost of goods/unit and s is the total value.

Now for a 3D space, we can find the distance between any 2 points on the space,

The formula is given for two point (p_1, q_1, r_1) and (p_2, q_2, r_2)

$$\text{The distance is given by } \text{distance} = \sqrt{(p_1 - p_2)^2 + (q_1 - q_2)^2 + (r_1 - r_2)^2}$$

Now the equation for a sphere is given for a center (u, v, w) and radius r

$$(x - u)^2 + (y - v)^2 + (z - w)^2 = r^2$$

Chapter 12 Comparative Statistics

A peek into General Cases:

For partial derivatives when we know $z = f(x, y)$ and c is a constant

$$F(x, y, z) = c \Rightarrow z'_x = -\frac{F'_x}{F'_z}, \quad z'_y = -\frac{F'_y}{F'_z} \text{ for } F'_z \neq 0$$

Now for a more general case,

$$F(x_1, x_2, \dots, x_n, z) = \frac{\partial F / \partial x_i}{\partial F / \partial z}, \text{ where } i = 1, 2, \dots, n$$

Understanding Linear Approximation

The Linear approximation to f(x) about (x_0, y_0) is given by the formula

$$f(x, y) \approx f(x_0, y_0) + f'_1(x_0, y_0)(x - x_0) + f'_2(x_0, y_0)(y - y_0)$$

Where, $f'_1(x_0, y_0)$, stands for first derivative of (x_0, y_0) in terms of x and $f'_2(x_0, y_0)$ stands for the first derivative of (x_0, y_0) , in terms of y.

Now one can generalize the above equation to re-write it for functions of several variables

Linear approximation to $z = f(x) = f(x_1, \dots, x_n)$ about $x^0 = (x_1^0, \dots, x_n^0)$

$$f(x) \approx f(x^0) + f'_1(x^0)(x - x^0) + \dots + f'_n(x^0)(x_n - x_n^0)$$

Again $f'_n(x^0)$ stands for the first derivative in terms of the nth variable in the equation