Chapter 13 Optimization

Just Applications

In this section just examples from the previous sections are given, just reading the examples should be sufficient, no new concepts are discussed however, going through each example is very important

What's Extreme-Value Theorem

The theorem is basically:
In a bounded closed plane P, if there is a function $f(x,y)$. Then within P there should be 2 sets of points, one where the value of $f(x,y)$is the maximum (u,v) and also where it is minimum (s,t)
$F(u,v) \leq f(x,y) \leq f(s,t)$

How to find Maximum and Minimum:

1. First step is to find the stationary points of the function f(x,y) in the interiors of the plane P,

2. First subdivide the boundary into smaller parts where largest and the smallest value can be found for the function f within that particular boundary easily

3. Now just substitute the values found in first and the second steps on the function f to find the minimum and the maximum

Multiple Variables:

For a point to be the maximum or minimum for a function f within the plane P there are two conditions

1. The point has to be within the plane P

2. The point when applied or substituted in the first derivative of the function should give the result as zero (0). Or in other word it should be a stationary point

If h(x) = F(g(x)), where g(x) is a function with one variable and h(x) is defined for plane P then

1. For the plane P, if a is the maximum or minimum for g(x) and also F is increasing then it is also the maximum and minimum for h(x) in the plane P

2. For the plane P, if a is the maximum or minimum for h(x) and F is strictly increasing only then is a the maximum or minimum for f in the plane P

Some theorems

$For\ f^*(r) = \ maximum\ for\ x\ in\ f(x,r)$

$and\ if\ x^*(r)\ is\ the\ result\ of\ x\ which\ maximum\ for\ f(x,r)\ then$

$$\frac{\partial f^*(r)}{\partial r_j} = \left[\frac{\partial f(x,r)}{\partial r_j}\right]_{x=x^*(r)}, where\ j = 1, \dots\dots, n$$

This can prove that partial derivatives exist

Chapter 14 Optimization Constrained

Variable and Constrains

A general way of writing a problem would be

$Max(min) f(x_1, \dots\dots.., x_n)\ \ when\ h(x_1, \dots \dots x_n) = a$

Using an example:

$\mathcal{L}\,(x,y,z) = (x-1)^2 + (y-2)^2 + (z-3)^2 - \gamma(x^2 + y^2 + z)$

Then first step is to find the first derivatives:

$\mathcal{L}_1'\,(x,y,z) = 2(x-1) \quad -2x\gamma = 0$

$\mathcal{L}_1'\,(x,y,z) = 2(y-2) \quad -2y\gamma = 0$

$\mathcal{L}_1'\,(x,y,z) = 2(z-3) \quad -\gamma = 0$

$x^2 + y^2 = -z$

Now solve the following to get the values of $x, y, z\ and\ \gamma$

In case of Constrains just subtract the sum of all the constrains from the equation and follow the same steps

What's Comparative Statics

Just one important thing to remember:

$$\frac{\partial f^*(r)}{\partial r_j} = \left[\frac{\partial f(x,r)}{\partial r_j}\right]_{x=x^*(r)}, where\ j = 1, \dots\dots, n$$

Non Linear Programming

For $\max f(x) subject\ to\ h_s(x) \le a,$ $where\ s = 1, \dots\dots\dots, n$ and

$x\ stands\ for\ (x_1, \dots\dots\dots, x_n)$

1. write the Lagrangian