$$\frac{\partial f^*(r)}{\partial r_j} = \left[\frac{\partial f(x,r)}{\partial r_j}\right]_{x=x^*(r)}, where\ j = 1, \dots \dots, n$$

This can prove that partial derivatives exist

Chapter 14 Optimization Constrained

Variable and Constrains

A general way of writing a problem would be

$Max(min) f(x_1, \dots \dots .., x_n)\ \ when\ h(x_1, \dots \dots x_n) = a$

Using an example:

$\mathcal{L}\,(x,y,z) = (x-1)^2 + \ (y-2)^2 + \ (z-3)^2 - \gamma(x^2 + y^2 + \ z)$

Then first step is to find the first derivatives:

$\mathcal{L}_1'\,(x,y,z) = 2(x-1) \quad -2x\gamma = 0$

$\mathcal{L}_1'\,(x,y,z) = 2(y-2) \quad -2y\gamma = 0$

$\mathcal{L}_1'\,(x,y,z) = 2(z-3) \quad -\gamma = 0$

$x^2 + y^2 = -z$

Now solve the following to get the values of $x, y, z\ and\ \gamma$

In case of Constrains just subtract the sum of all the constrains from the equation and follow the same steps

What's Comparative Statics

Just one important thing to remember:

$$\frac{\partial f^*(r)}{\partial r_j} = \left[\frac{\partial f(x,r)}{\partial r_j}\right]_{x=x^*(r)}, where\ j = 1, \dots \dots, n$$

Non Linear Programming

For $\max f(x) subject\ to\ h_s(x) \leq a,$ $where\ s = 1, \dots \dots \dots, n$ and

$x\ stands\ for\ (x_1, \dots \dots \dots, x_n)$

1. write the Lagrangian

$$\mathcal{L}(x) = f(x) - \sum_{s=1}^{n} \gamma_s (h_s(x) - a_s)$$

2. Equate all the partial derivatives to 0

$$\frac{\partial \mathcal{L}(x)}{\partial x_j} = \frac{\partial f(x)}{\partial x_j} - \sum_{s=1}^{n} \gamma_s \frac{\partial h_s(x)}{\partial x_j} = 0,$$ where j= 1,………,m

3. Apply the complementary slackness conditions:

$$\gamma_s \geq 0, if\ h_s(x) < a\ then\ \gamma_s = 0$$

4. Satisfy the constrain

$$h_s(x) \leq a_s$$

We should find all vectors x for which all the above are sure, then it could be considered as a possible solution.