Chapter 2 Review of basic mathematics continued- Equations:

Solving ‘easy’ equations:

Ex. 1: $4x + 5 = 12 - 3x$ First you isolate x, and then you simply solve for x.

$4x - 3x + 5 = 12$

$7x + 5 = 12$

$7x = 17$

$X = 17/7$

In some cases you will have to find the lowest common denominator in order to solve for x:

Ex.2: $\frac{4x+5}{x-2} - \frac{9}{x^2-4} = \frac{12}{x+2}$

We can’t subtract the fractions since the denominators the fractions are not the same- so first we have to find the lowest common denominator.

The easiest is to first find the factor pairs of $x^2 - 4$, which are: $(x-2)$ and $(x+2)$, and then we find that the lowest common denominator must be $(x-2)(x+2)$. Finally, you multiply both sides by the lowest common denominator:

$\frac{4x+5}{x-2} \cdot (x+2)(x-2) - \frac{9}{x^2-4}(x+2)(x-2)$
$= \frac{12}{x+2}(x+2)(x-2)$

$= \frac{4x+5(x+2)-9}{(x-2)(x+2)} = \frac{12(x-2)}{(x-2)(x+2)}$

$4x^2 + 13x + 1 = 12x - 4$

$4x^2 + x + 5 = 0$

At this point you would use the quadratic formula to solve for x. However, we will discuss this later on, this example is only meant to illustrate how to get rid of fractions by multiplying by the lowest common denominator.

Linear equations and a few examples of its application in economics:

The general linear equation is : $y = ax + b$

Where a and b are called parameters, which simply means that they can take on different values.

A few examples of linear equations in economics are:

1. $Y = C + I$ A country's GDP ( ) equals consumption (C) plus investment (I).
2. $C = a + bY$ Consumption (C) is a linear function of GDP (Y)

Basic Rules of Quadratic Equations:

1. For a quadratic in the form $ax^2 + bx + c = 0,\ where\ b^2 - 4ab \geq 0\ and\ a \neq 0$

   $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ (Quadratic formula)

2. $ax^2 + bx + c = a(x - r)(x - s), if\ ax^2 + bx + c = 0\ and\ r, s\ are\ solutions\,.$
3. $r + s = -\frac{b}{a}\ and\ r \times s = \frac{c}{a}\ for\ ax^2 + bx + c = 0\ where\ r\ and\ s\ are\ solutions.$

Example: $p^2 - 10p + 21 = 0$

Without using the quadratic formula: $p^2 - 10p + 21 = 0$

From rule 3, let r and s be the solutions, then $r + s = -\frac{-10}{1}\ and\ r \times s = \frac{21}{1}$, so r and s could be 7 and 3, as $7 + 3 = 10, 7x3 = 21.$

Therefore: Using rule two the equation can be written as, $1(p - 3)(p - 7) = 0$, where 3 and 7 are the solutions.

Using Quadratic formula: In the formula; $a = 1, b = -10$ and $c = 21,$

Thus, $x = \frac{10 \pm \sqrt{100 - 4 \times 1 \times 21}}{2} = \frac{10 \pm \sqrt{16}}{2} = \frac{10 \pm 4}{2}$

$= 5 + 2\ or\ 5 - 2, therfore\ 7\ and\ 3\ are\ the\ solutions$

Linear Equations with 2 variables:

1. Substitution Method: First find the value of one variable in terms of the other, then substitute the value into the second equation.

Example: $4x - 3y = 7$ and $2x + 5y = 23$ solve for x and y.

First using first equation, $4x - 3y = 7 \Rightarrow 4x = 7 + 3y, x = \frac{7+3y}{4}$

Now substitute the value of x in terms of y in equation $2, 2(\frac{7+3y}{4}) + 5y = 23$

((7+3y+10y)/2) = 23, 7+13y = 46, 13y= 39, y=3 and thus x = $(\frac{7+3\times3}{4})$ = 4

2. Elimination Method: We multiply both the equation with a constant such that when equation 1 is subtracted from equation 2, one variable is removed. Thus leaving a simple linear equation.

Example: $4x - 3y = 7$ and $2x + 5y = 23$ solve for x and y.

Multiplying equation 1 with 1 = > $4x - 3y = 7$

Multiplying equation 2 with 2 => $-4x + 10y = 46$

$$0x - 13y = -39,$$

Therefore, $y = 3$ and now substitute the value in equation 1 or 2 to get $x = 4.$

Final note:
There is one very important fact one should remember whenever solving equations, and that is:
When you multiply two or more factors, their product can only be zero if at least one of their factors is zero.
Example: $x(x+3)(x-2) = 0$ Whenever an equation is set up this way, the first solution that must cross your mind is that $x = 0.$ (and of course, we will also find that $x = -3$ and $x = 2$ are the other solutions).

Summation Notation: $\mathrm{P_a} + \mathrm{P_{a+1}} + \mathrm{P_{a+2}} \ldots \ldots\ \mathrm{P_n} = \sum_{k=a}^{n} \mathrm{P_a}$
The K= a, indicates the starting number of the sequence, n= number of times.

Chapter 3 Basic Notations

Economic Application: Price Indices (Inflation) for a group of goods

$$\frac{\textit{Total cost of goods in final year}}{\textit{Total cost of goods in Initial year}} = \frac{\sum_{k=1}^{m} \mathrm{P_F^{(k)} Q^{(k)}}}{\sum_{k=1}^{m} \mathrm{P_I^{(k)} Q^{(k)}}} \times 100 = \text{Price Index}$$

Where, $\mathrm{Q^{(k)}}$ = Number of gook K,

$\mathrm{P_I^{(k)}}$= Price/good k in Initial/first year

$\mathrm{P_F^{(k)}}$= Price/good k in final year

Laspeyres Price Index: When the quantity consumed is based on the initial year f or the above formula.
Paashe Price Index: When the quantity consumed is based on the final year f or the above formula.

Properties:

1. $\sum_{k=1}^{n}(\mathrm{s_k} + \mathrm{r_k}) = \sum_{k=1}^{n} \mathrm{s_k} + \sum_{k=1}^{n} \mathrm{r_k}$
2. $\mathrm{r} \sum_{k=1}^{n} \mathrm{s_k} = \sum_{k=1}^{n} \mathrm{r} \times \mathrm{s_k}$
3. $\sum_{k=1}^{n}(\mathrm{s_k} + \mathrm{r}\ \ ) = \sum_{k=1}^{n} \mathrm{s_k} + \mathrm{nr}$
4. $\sum_{k=1}^{n} \mathrm{K}^2 = \frac{1}{6} n(n+1)(2n+1)$
5. $\sum_{k=1}^{n} \mathrm{K}^3 = \left[\sum_{k=1}^{n} \mathrm{K}\ \right]^2$
6. $(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^k a^{n-k}, \text{where } \binom{\mathrm{n}}{\mathrm{k}} = \frac{\mathrm{n(n-1)(n-2) \ldots\ldots (n-m+1)}}{\mathrm{m!}}$

Example: $\binom{7}{3} = \frac{7 \times 6 \times 5}{3 \times 2 \times 1} = 7 \times 5 = 35$