7. $\sum_{k=1}^{n} s_{k1} + \sum_{k=1}^{n} s_{k2} + \ldots + \sum_{k=1}^{n} s_{kb} = \sum_{r=1}^{n}[\sum_{k=1}^{n} s_{kr}]$

$$\textbf{Example:} \quad \sum_{r=1}^{2}\left[\sum_{k=1}^{3}(2r+4k)\right] = \sum_{r=1}^{2}[(2r+4) + (2r+8) + (2r+12)]$$

$$= \sum_{r=1}^{2}[6r+24] = (6+24) + (12+24) = 66$$

Just Logic:

Implication - symbol: '=>' or '<=', and the pointer shows the direct the logic holds true,

1. Logical equivalence – symbol '⇔' this means that the statement on right and left are true if even one of them is true.
2. The term 'or' in mathematics implies that in the case 'a' or 'b', both terms 'a' and 'b' hold true.
3. Necessary condition - b=> a, For example, 'the brakes are working (a)' is necessary condition for saying 'the bicycle is in good condition (b)'
4. Sufficient Condition - a=>b, For example, 'the brakes are working (a)' is sufficient condition for saying 'the bicycle is in good condition (b)' is false, as the brakes could be working but the wheels could be missing. But 'heart is beating' is sufficient condition for saying 'the person is alive'. Because, even if all the other systems are dead, the person is still considered to be alive as long as his heart is beating/or made to beat.
5. Deductive Reasoning: Used in Mathematics, reasoning is based on logic
6. Inductive Reasoning: Coming to general assumptions based on few situations or cases.
7. Venn Diagram:

$W \subseteq X$

Fig.4:

W is a part of X, all values of W are values of X

The non-colored parts- Fig5

The non-colored part- Fig.6

The non-colored part-Fig.7

Chapter 4 Functions

Defining terms:

Independent variable : a variable that causes change in other variables.

Dependent variable : a variable whose value depends on the value of other variables.

How does this relate to functions?
$y = f(x)$
In this case, x is the independent variable, and y is the dependent variable- the value of y will depend on the value of x.
Domain: The domain of the function f (shown above) is the set of all possible values for x (the independent variable).
Range : The set of the corresponding values of y (the dependent value)
To conclude, the range of a function depends on its domain.
A small note on the function notation $f(x)$
The f can simply be replaced by a different letter, for instance you can have a question with $R(x)$, however $R(x)$ remains a function, and thus will also have the following property:
The natural domain of the function f is the set of all real numbers--- and these real numbers must give only one y value for each x value.
Example: $f(x) = x^3 + 4$ passes the vertical line test (since the vertical lines only touch the graph once- each x has only one y.
Fig.8

Graphing functions:

The xy plane (also called coordinate system):
Fig.9
Important functions of whose graphs you should know the general shape:

$y = x$ $\quad$ $y = x^2$ $\quad$ $y = x^3$

$y = \sqrt{x}$ $\quad$ $y = \frac{1}{x}$ $\quad$ $y = |x|$

f(x)=ax +b

A linear function is written as: $f(x) = ax + b$, where a and b are constants.
Properties of $f(x) = ax + b$ are:

- Its graph is a straight line
- a represents the slope
- b is the y-intercept.

Calculating the slope:
Slope $a = \frac{y_2 - y_1}{x_2 - x_1}$, where x_1 cannot equal x_2- since otherwise the denominator would equal zero, and we cannot divide by zero.
Different methods of finding the equation: y=ax+b

1. **Point slope formula:** $y - y_1 = a(x - x_1)$

Use: If given a point and slope in a question.
Example: A line passes through (2, 4) and its slope is 4. Find the equation of the line.
$(x_1, y_1) = (2,4)$ and $a = 4$. Thus, $y - 4 = 4(x - 2)$ $\quad$ $y = 4x - 4$

2. **Point-point formula:**
Use if given two points:

Example: A line passes through (-3, -4) and (2,1). Find the equation of the line. $a = \frac{1-(-4)}{2-(-3)}$

$a = \frac{2}{3}\ y - (-4) = \frac{2}{3}[x - (-3)] \qquad 2 = \frac{2}{3}x - y$

You also need to know the general equation for a straight line, which is : $Ax + By + C = 0$, where A simply equals the slope(which is a), but note that $B = -1$ and $C = b$.

3. **Graphing:**
Another method used with linear equation is graphing- you can graph, for instance, two equations and then find their intersection point.

Graphing linear inequalities:
Example: Sketch the graph of $2x + y \leq 18$
First you simply graph the equation: $y = -2x + 18$, and then shade the appropriate area:
Fig.16

Linear models and their applications in economics:
1. The consumption function: $C = a + bY$

In this case, C represents consumption, b represents the marginal propensity to consume (for instance, when your income increases, how much are you going to consume of that increase in income?) and Y represents (national) income.

2. Supply and demand: basically you can have an equation for demand and an equation for supply (both equations will include a P for price). If you want to find the equilibrium price and the equilibrium quantity, you set the demand and supply equation equal to eachother, and then you simply solve for P to find the equilibrium price, and by subsituting the equilibrium price back into one of the equations, you can find the equilbirum quantity as well.

Example: $C = 100 + 20P,\ \ D = 80 + 40P$, thus if you set both equations equal to eachother, you find that the equilibirum price equals 1. If you substitute 1 back into one of the equations, for instance $D = 80 + 40(1)$, you find that the equilibrium quantity will be 120.

Quadratic Equations:

The general quadratic equation format is $f(x) = ax^2 + bx + c$, the graph on the equation is called a parabola.
Fig.17: $a < 0, b^2 > \ 4ac$

Fig.18: $a > 0, b^2 < 4ac$
For the equation $f(x) = ax^2 + bx + c$, the minimum or the maximum is given by
$c - \frac{b^2}{4a}$ at $x = \ -\frac{b}{a}$

1. If a>0, then it is the minimum value,
2. If a<0, then it is the maximum value,

General Polynomial

A general polynomial is any equation in form of:
$F(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \dots + a_1 x^1 + a_0$
When F(x) has a factor (x-k), F(k)=0
Division of Polynomials:
Example$(x^4 + 2x^3 + 2x^2 + 8) \div (x + 2)$, thus the questioned is $x^3 + 2x - 4$, and the remainder is 17.

```
x + 2 |  x^4 + 2x^3 + 2x^2 + 9 | x^3 + 2x - 4
        (-)x^4 (-)2x^3
        ----------------------
                  + 2x^2      + 9
                  (-)2x^2 (-) 4x + 0
                  ------------------
                          - 4x + 9
                        (-)4x (+) 8
                        -----------
                               17
```

Functions and Powers:

1. $f(x) = Bk^n$ $(k > o, n \text{ and } B \text{ are any constants})$
2. *Shifts in the graph of* k^n *for different values of* n.

   (1) $\Rightarrow f(a) = a^3$

   (2) $\Rightarrow f(a) = a^2$

   (3) $\Rightarrow f(a) = a$

   (4) $\Rightarrow f(a) = a^{\frac{1}{2}}$

   (5) $\Rightarrow f(a) = a^{\frac{1}{3}}$

Functions and exponents:

An exponential function is a function where the variable is the power, $f(k) = Bb^k$,
The graph of the function $f(k) = Bb^k, \text{when } b > 1.$
Fig.19
The graph of the function $f(k) = Bb^k, \text{when } 0 < b < 1$
Fig.20
Application:

1. Population Growth: $P(t) = \text{population in base year} \times (1 + \frac{\text{rate of growth}}{100})^{\text{time in years}}$

2. Compound Interest: A = $A = P\left[1 \pm \frac{r}{100}\right]^t$ where A = Total Amount, P = Initial amount, r = rate of change/interest rate (% per year), t = Time in years, + when the rate in increasing, - when the rate in decreasing.

3. Continuous Depreciation: When the value of the asset decreases with the same percentage each year then it is called continuous depreciation,

   $$V(t) = P_i\left(1 - \frac{r}{100}\right)^t,$$

   $where\ V(t) is\ value\ of\ the\ assest\ on\ t\ year,$

   $P_i\ is\ the\ purchased\ price\ of\ the\ asset$

   $r\ is\ the\ rate\ of\ depreciation$

   $t\ is\ the\ time\ in\ years$

Functions and logarithms:

The general form is $e^{lnx} = x, for\ all\ positive\ values\ of\ x$

Properties of the natural logarithm:

1. $\ln(xy) = lnx + lny$
2. $ln\frac{x}{y} = lnx - lny$
3. $lnx^p = plnx$
4. $ln1 = 0$
5. $lne = 1$
6. $x = e^{lnx}, x > 0$
7. $lne^x = x$

These rules also apply to logarithms with bases other than e, for instance $log_a(xy) = log_a x + log_a y$, is the same as rule 1.

Chapter 5 Functions continued

Shifting the graph of $y = f(x)$

1) $y = f(x) \longrightarrow y = f(x) + d$, the graph moves upwards by d units if d is positive, and the graph will move down by d units if d is negative.

2) $y = f(x) \longrightarrow y = f(x + d)$, the graph moves d units to the left if d is positive, and the grpah will move to the right by d units, if d is negative.