

2. Compound Interest: $A = P \left[1 \pm \frac{r}{100} \right]^t$ where A = Total Amount, P = Initial amount, r = rate of change/interest rate (% per year), t = Time in years, + when the rate is increasing, - when the rate is decreasing.

3. Continuous Depreciation: When the value of the asset decreases with the same percentage each year then it is called continuous depreciation,

$$V(t) = P_t \left(1 - \frac{r}{100} \right)^t,$$

where $V(t)$ is value of the asset on t year,

P_t is the purchased price of the asset

r is the rate of depreciation

t is the time in years

Functions and logarithms:

The general form is $e^{lnx} = x$, for all positive values of x

Properties of the natural logarithm:

1. $\ln(xy) = \ln x + \ln y$

2. $\ln \frac{x}{y} = \ln x - \ln y$

3. $\ln x^p = p \ln x$

4. $\ln 1 = 0$

5. $\ln e = 1$

6. $x = e^{lnx}, x > 0$

7. $\ln e^x = x$

These rules also apply to logarithms with bases other than e , for instance

$\log_a(xy) = \log_a x + \log_a y$, is the same as rule 1.

Chapter 5 Functions continued

Shifting the graph of $y = f(x)$

- 1) $y = f(x) \longrightarrow y = f(x) + d$, the graph moves upwards by d units if d is positive, and the graph will move down by d units if d is negative.
- 2) $y = f(x) \longrightarrow y = f(x + d)$, the graph moves d units to the left if d is positive, and the graph will move to the right by d units, if d is negative.

- 3) $y = f(x) \longrightarrow y = df(x)$, the graph is stretched vertically if d is positive, and if d is negative, the graph will be stretched vertically and will be reflected about the x-axis.
- 4) $y = f(x) \longrightarrow y = f(-x)$, the graph is reflected about the y-axis.

Small advice: If you somehow forget these rules during the exam, just make a table, and substitute some values in for x , and see what values of y it yields, and then plot the points on a graph and you will find how the graph shifts.

Introducing different types and properties of functions:

Defining product and quotient:

Product: if $h(x) = f(x) \cdot g(x)$, then h is the product of f and g .

Quotient: if $h(x) = \frac{f(x)}{g(x)}$ then h is the quotient of f and g .

Composite functions:

One gets a composite function if you put the output of one function in the input of the another function.

Ex. $f(x) = x^2 - 4$, and $g(x) = x - 2$, find $f(g(x))$.

$$f(g(x)) = (x - 2)^2 - 4$$

$$f(g(x)) = x^2 - 4x$$

Note : $y = f(g(x))$ can also be written as $f \circ g$, and it is read as: f of g .

Symmetry with odd and even functions:

Graphs.

Other Functions:

Inverse function:

A function that is simply the reverse of the given function: $f(x) = y$ then $f^{-1}(y) = x$.

Ex. The demand for crisps is described by the following function: $D(p) = 20 + 30p$.

However, as a producer of the crisps I don't want to know the demand for a specific price, but I want to decide on a certain output, and see what the price of that output will be.

Thus, I will want to find the inverse of the demand function given earlier:

$$D(p) = 20 + 30p$$

$$D = 20 + 30p \text{ (step 1: rewrite the function as an equation)}$$

$$p = \frac{D-20}{30} \text{ (step 2: solve for } p \text{)}$$

$$D^{-1}(p) = \frac{p-20}{30} \text{ (switch } p \text{ and } D \text{ back and you have your answer)}$$

One-to-one:

One-to-one simply means that the equation passes the vertical line test, mentioned earlier. Thus any function will be one-to-one, and will therefore have an inverse. If it is not one-to-one, it is not a function, and it will not have an inverse.

Domain and range of an inverse:

The domain of the inverse is the range of the original function, and the range of the inverse is the domain of the original function.

Important note on the graphs of inverse functions:

The inverse function will always be reflected over $y = x$.

($y = x$ is the line that runs through the origin of the graph)