$f(x) = e^u \to f'(x) = u' \cdot e^u$

Logarithmic functions and differentiation:

The derivative of lnx:

$f(x) = lnx \to f'(x) = \frac{1}{x}$

Ex. Find $f'(x)$ when $f(x) = lnx + x^2$

$f'(x) = \frac{1}{x} + 2x$

Chapter 7 Application of Derivatives:

Application of Derivatives:

Implicit Differentiation:

1. Implicit Differentiation is used when the function is not in the form a= f(b), but the variables x and y could be on either side of the equation,
2. Instead of getting the equations to the form a=f(b), just take derivative on both sides (REMEMBER: re-write ‘a’ as f(b) considering ‘a’ to be a function of ‘b’)
3. Solve the equation for y’

Example:

$2a^2 + 4ab = 2b^2$

$2a^2 + 4af(b) = 2[f(a)]^2$

(Remember: use chain and product rule where applicable)

$2 \times 2 \times a + 4a^0 f(a) + 4af'(a) = 2 \times 2 \times f(a) \times f'(a)$

$4a + 4f(a) + 4af'(a) = 4f(a)f'(a)$

$4a + 4f(a) = f'(a)[4f(a) - 4a]$

$f'(a) = \frac{4a + 4f(a)}{4f(a) - 4a} = \frac{4a + 4b}{4b - 4a} = b'$

Inverse and Differentiation

If a function is differentiable, and it is strictly increasing or decreasing then for that interval function will have an inverse.

$for\ a\ function\ b = f(a), \qquad h'(b_i) = \frac{1}{f'(a_i)}$

For Example: $Find\ the\ inverse\ of\ the\ equation\ f(x) = e^x$

$y = e^x = f(x)$

$x = g(y) = lny$

$g'(y) = \frac{1}{f'(x)}\ from\ the\ equation\ \ given\ above$

$$\frac{1}{y} = \frac{1}{e^x} \qquad \textit{therefore, the inverse function is } e^x = y$$

Understanding Linear Approximations:

When there is a complex function then we can sometime replace it with a linear function with a similar graph.
For a value $a = u$
$f(a) \approx f(u) + f'(u)(a - u)$

Example: Find the approximate value of $(1.003)^{60}$
We can consider this equation to be $f(x) = x^{60}, where\ x = 1.003,$
$Now\ let\ us\ assume\ x = 1.003 \approx a = 1$
Using the formula,
$f(1.003^{60}) \approx f(1) + f'(1^{60})(1.003 - 1)$
First we solve; $f'(1^{60}) = 60 \times 1^{59}$ = 60
Therefore,
$f(1.003^{60}) \approx 1 + 60(0.003) = 1.018$

Reminder Rules (For Differentiation)

1. $d(uf + vg) = udf + vdg$

$(where\ d\ stands\ for\ differentiate, \quad u, v\ are\ constants, \quad f, g\ are\ functions)$

2. $d(fg) = gdf + fdg$

3. $d\left(\frac{f}{g}\right) = \frac{gdf - fdg}{g^2}$

Understanding Polynomial Approximations:

When there is a complex polynomial then we can sometime replace it to make calculations easier
Quadratic Approximation:
For a value $a = u$

$$f(a) \approx f(u) + f'(u)(a - u) + \frac{1}{2} f''(u)(a - u)^2$$

Other Polynomials with Higher Order

$$f(a) \approx f(u) + \frac{f'(u)}{1!}(a - u) + \frac{f''(u)}{2!}(a - u)\text{^}2 + \ldots\ldots\ldots + \frac{f^n(u)}{n!}(a - u)^n$$

Application of Differentiation:

Price Elasticity of Demand: The symbol used for price elasticity is $EL_p\ D(p)$. There is a simple formula which by definition means that for any function of demand with respect to price D(p), the price elasticity is

t

$$EL_p\, D(p) = \frac{p}{D(p)} \times \frac{dD(p)}{dp}$$

An easier formula which can be derived from the above formula is:

$EL_x f(x) = \frac{x}{f(x)} \times f'(x)$ …. General formula for elasticity

Example: An economist has figured out that relation between demand and price of beer in Rotterdam is given by the function D$(p) = \frac{p-1}{p+1}$, Find the Price elasticity of Beer in Rotterdam

$EL_p D(p) = \frac{p}{D(p)} \times D'(p) \ldots .\mathit{from\ the}$ General formula

$EL_p D(p) = \frac{p\,(p+1)}{(p-1)} \times \frac{(p+1)-(p-1)}{(p+1)^2}$ ….. using quotient rule

$EL_p D(p) = \frac{p}{(p-1)} \times \frac{2}{p+1}$

$EL_p D(p) = \frac{2p}{(p-1)^2}$ …….. is the Price elasticity of Beer in Rotterdam

Understanding Continuity:

If a function is differentiable for a certain interval then the function is also continues for that set of interval.

For a function to be continues the following condition has to be true.

$\lim_{n\to a^-} f(x) = \lim_{n\to a^+} f(x)$,
where a^- means values of a approaching from the left of a in the number line
a^+ means values of a approaching from the left of a in the number line

Example: Find out if the function is continues or not,
$f(x) = 3x - 2\ when\ x \leq 2$
$f(x) = -x + 6\ when\ x > 2$

$\lim_{n\to 2^-} f(x) = 3(2) - 2 = 4 \ldots..\ when\ x \leq 2$

$\lim_{n\to 2^+} f(x) = -2 + 6 = 4 \ldots \ldots when\ x > 2$

Since, $\lim_{n\to 2^-} f(x) = \lim_{n\to 2^+} f(x)$ *is true it is a continues function*