Chapter 8 Optimal Points

Maximum and Minimum points

- When $f'(x) = 0$, then the point at which the derivative equals zero is called a stationary point.
- There are different types of stationary points, for now we need to know the maximum and minimum points:
- Minimum point: concave up.
- Maximum point: concave down.

Testing the nature of stationary points:
We can find the nature of stationary points by, first of all, using the first derivative test, which is illustrated in the example below (economic applications of the derivative).

Economic applications of the derivative:

Ex. When the price of a product is *p*, the revenue can be found by $R = 5p^2 + 20p + 16$. What price maximizes the revenue?
$Step\ 1: Find\ the\ first\ derivative \rightarrow R' = -10p + 20$
Step 2: $Set\ the\ first\ derivative\ equal\ to\ zero\ and\ solve\ for\ p:$
$0 = -10p + 20$
$-20 = -10p$
$2 = p$
Before we can conclude whether revenue is maximized at 2, we need to check whether 2 is indeed a maximum point- the easiest way is to find the second derivative, and if the second derivative is negative for 2, then 2 is indeed a maximum point. (the second derivative will be positive if 2 is a minimum point).
$R'' = -10$ Thus, 2 is indeed the maximum point.

Extrema:

In this chapter they discuss the extreme points (maximum and minimum points) on a closed interval. This closed interval will be indicated by the following brackets: $[2,5]$.
All you have to do to find the extreme points is the following:
Ex. Find the maximum and minimum values of the function $f(x) = 2x^2 - 4x + 5$. $x \in [0,5]$
Step 1: Find the stationary points:
$f'(x) = 4x - 4$
$0 = 4x - 4$
$4 = 4x$
$x = 1$
$f(1) = 2(1)^2 - 4(1) + 5$
$f(1) = 3$
Step 2: Find the y-values of the end points of the interval:
$f(0) = 2(0)^2 - 4(0) + 5$
$f(0) = 5$
$f(5) = 2(5)^2 - 4(5) + 5$
$f(5) = 35$

So looking at all three points (0,5) (1,3) (5,35) we can see that (1,3) is the minimum value.

Local maximums and minimums
Note that we already explained the first and second derivative test earlier-these tests are used to find the local maximums and minimums.

Points of inflection:

At an inflection point, the graph changes concavity- for instance, from concave down to concave up.
If the second derivative equals zero, then the point is an inflection point!
Review of convex and concave:
$f''(x) < 0 \; for\; all\; x \in (c,d), f(x) is\; concave\; in\; (c,d)$
$f''(x) > 0 \; for\; all\; x \in (c,d), f(x) is\; convex\; in\; (c,d)$

Chapter 9 Integrals

Family of all anti-derivatives of f(x):

The indefinite integral of a function $f(x)$ is written as $\int f(x)dx$, and it is the family of all anti-derivatives of a function.
You can find the indefinite integral the following way:
Ex. If $f = 4x^2$ and thus the derivative is $8x$, then the indefinite integral is $4x^2 + c$.
Thus, to integrate you first add a number to the exponent on x and you divide the number in front of x by this new exponent, this rule can be written as: $\frac{1}{n+1}x^{n+1} + C$.
But why do we have a c at the end? In order to explain this, I will use an example:
Ex. If $f(x) = x^2 + 4x + 2$, then the derivative would be $f'(x) = 2x + 4$. By integration you would find: $x^2 + 4x$. However, in integration questions the original function is never given, and thus you would never know that there was a 2 at the end of the function, since it disappears in the derivative. That's why we add the c at the end for the indefinite integral; the c represents any number that could have been behind $x^2 + 4x$.

Important note:
Note that the rule for integration, introduced earlier, was: $\int x^n \, dx = \frac{1}{n+1}x^{n+1} + C$, where $n \neq 1$. This exception $n \neq 1$ is very important-since we have a separate rule for functions where $n = 1$. The rule is the following: $\int \frac{1}{x} dx = ln|x| + C$.
More formulas:
$$\int e^{ax} \, dx = \frac{1}{a}e^{ax} + C, where\; (a \neq 0)$$
$$\int a^x dx = \frac{1}{lna}a^x + C, where\; (a > 0\; and\; a \neq 1)$$
…… And some more rules:
$$\int af(x)dx = a \int f(x)dx \;\; (a\; is\; a\; constant)$$

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