So looking at all three points (0,5) (1,3) (5,35) we can see that (1,3) is the minimum value.

Local maximums and minimums
Note that we already explained the first and second derivative test earlier-these tests are used to find the local maximums and minimums.

Points of inflection:

At an inflection point, the graph changes concavity- for instance, from concave down to concave up.
If the second derivative equals zero, then the point is an inflection point!
Review of convex and concave:
$f''(x) < 0 \; for\; all\; x \in (c,d), f(x) is\; concave\; in\; (c,d)$
$f''(x) > 0 \; for\; all\; x \in (c,d), f(x) is\; convex\; in\; (c,d)$

Chapter 9 Integrals

Family of all anti-derivatives of f(x):

The indefinite integral of a function $f(x)$ is written as $\int f(x)dx$, and it is the family of all anti-derivatives of a function.
You can find the indefinite integral the following way:
Ex. If $f = 4x^2$ and thus the derivative is $8x$, then the indefinite integral is $4x^2 + c$.
Thus, to integrate you first add a number to the exponent on x and you divide the number in front of x by this new exponent, this rule can be written as: $\frac{1}{n+1}x^{n+1} + C$.
But why do we have a c at the end? In order to explain this, I will use an example:
Ex. If $f(x) = x^2 + 4x + 2$, then the derivative would be $f'(x) = 2x + 4$. By integration you would find: $x^2 + 4x$. However, in integration questions the original function is never given, and thus you would never know that there was a 2 at the end of the function, since it disappears in the derivative. That's why we add the c at the end for the indefinite integral; the c represents any number that could have been behind $x^2 + 4x$.

Important note:
Note that the rule for integration, introduced earlier, was: $\int x^n\, dx = \frac{1}{n+1}x^{n+1} + C$, where $n \neq 1$. This exception $n \neq 1$ is very important-since we have a separate rule for functions where $n = 1$. The rule is the following: $\int \frac{1}{x}dx = ln|x| + C$.
More formulas:
$$\int e^{ax}\, dx = \frac{1}{a}e^{ax} + C, where\; (a \neq 0)$$
$$\int a^x dx = \frac{1}{lna}a^x + C, where\; (a > 0\; and\; a \neq 1)$$
…… And some more rules:
$$\int af(x)dx = a\int f(x)dx \quad (a\; is\; a\; constant)$$

$$\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx$$

Definite Integration:

The exact area under a curve is given by the definite integral, which is defined the following way: $\int_a^b f(x)dx = |_a^b\ F(x) = F(b) - F(a), where\ F'(x) = f(x) for\ all\ x \in (a,b)$

Ex. Evaluate $\int_2^4 (x^2 - 4x)dx$.

$=|_2^4 \left(\frac{1}{3}x^3 - 2x^2\right)$

$= [\frac{1}{3}(4)^3 - 2(4)^2] - [\frac{1}{3}(2)^3 - 2(2)^2]$

$= \frac{16}{3} \quad - \frac{16}{3}$

= 0.

Thus, there is no area under the curve.

However, what is the area if $f(x)$ is negative?

In that case the area you are finding is below the x-axis- however, note that you are still finding an area- there is simply a negative side in front of the integral. The area under the x-axis is simply subtracted from the total area.

Properties:

The following are properties of the definite integral:

$$\int_a^b f(x)dx = -\int_b^a f(x)dx$$

$$\int_a^a f(x)dx = 0$$

$\int_a^b \propto f(x)dx = \propto \int_a^b f(x)dx$, where $\propto is\ just\ some\ random\ number$

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

More rules…

1. The derivative of the definite integral, with respect to the upper limit of integration is equal to the integrand (the expression you are integrating) at that limit:

   $\frac{d}{dz}\int_a^z f(x)dx = F'(z) = f(z)$

2. The derivative of a definite integral with respect to the lower limit of integration is equal to minus the integrand evaluated at the limit:

   $$\frac{d}{dt}\int_z^w f(x)dx = -F'(z) = -f(z)$$

Finally we have an important note: you can integrate continuous functions!

Understanding Integration using Parts

To integrate an equation by parts we need to first re-write the equation in the form of $f(x) \times h'(x)dx$, or

$a\ function\ with\ variable\ x \times derivative\ of\ another\ function\ in\ terms\ of\ x$ then the formula used would be:

$$\int f(x)h'(x)dx = f(x)h(x) - \int f'(x)dxh(x)$$

Example: solve $\int 2x^4$

So it can be re-written as $\int (x)^3 \times 2x$

Then $f(x) = x^3 and\ h(x) = x^2, since\ 2x\ is\ the\ derivative\ of\ x^2$

$$\int x^3 \times 2x(dx) = x^3 \times x^2 - \int 3x^2(dx) \times x^2$$

This also implies:

$$\int_q^p f(x)h'(x)dx = [_q^p\ f(x)h(x) - \int_q^p f'(x)dxh(x)$$

Understanding Integration using Substitution

To integrate using substitution the equation should be re-written in the form of a composite function $f\big(h(x)\big) \times h'(x)dx$

$$\int f(h(x))h'(x)dx = f(a)da, where\ a = h(x)$$

Integrals with infinite intervals

Just remember: when $p \geq q$ then to integrate the equation can be written as

Either $\int_q^\infty f(x)dx = \lim_{p\to\infty} \int_q^p f(x)dx$

Or $\int_\infty^p f(x)dx = \lim_{q\to-\infty} \int_q^p f(x)dx$

Both the equations will give the same answer but are looking at the positive and negative infinity respectively

Reviewing Differential Equations

It is important to understand that $\dot{x}(n) = rx(n)\ \ for\ all\ values\ of\ \ n\ and\ r\ is\ a\ constant$

Then we can use this knowledge along with $x(0) = p$ to say that

$x(n) = pe^{rn}$

When there is a maximum limit or carrying capacity of L is involved we should use

$\dot{x}(n) = a(L - x(n))$, a is a constant

Again if $x(0) = p$, then $x(n) = L - (L-p)e^{-an}$

For a logistic function when

$$\dot{x}(n) = ax(n)(1 - \frac{x(n)}{L})$$

And we know that $x(0) = p$, then

$$X(n) = \frac{K}{1 + \frac{k-p}{p}e^{-rn}}$$

Also Remember:

1. $\dot{x} = px\ for\ all\ values\ of\ n\ ,$

$x = Se^{pn}, where\ S\ is\ a\ constant$

2. $\dot{x} + px = q\ for\ all\ values\ of\ n,$

$$x = Se^{-pn} + \frac{q}{p}, where\ S\ is\ a\ constant$$

3. $\dot{x} + px = qx^2\ for\ all\ values\ of\ n,$

$$x = \frac{p}{q - Se^{pn}}\ where\ S\ is\ a\ constant$$

Chapter 10 Rate of Interest and Values

Repayment of Mortgage

Let us look at a sample question which would sound like, A person has taken a mortgage for € A that he will pay over t installments at the rate of r% compounded annually. How much will he pay per installment?

From the previous section, Let p be the amount for each installment so

$$\frac{p}{r} \times 100 \left[1 - \frac{1}{(1 + \frac{r}{100})^n}\right] = A$$

Example: Let us use the example of the next door neighbor who has taken a mortgage of € 100000 that he will pay over 4 installments at the rate of 20% compounded annually. How much will he pay per installment?

$$\frac{p}{20} \times 100 \left[1 - \frac{1}{\left(1 + \frac{20}{100}\right)^4}\right] = 100000$$

$$p \times 5 \left[1 - \frac{1}{(1.20)^4}\right] = 100000$$

$$p = \frac{100000}{5} \times 0.5177$$

$$p = 10354$$

Now to find the amount per installment or p we can just use:

$$p = \frac{rA}{1 - (1 + r)^{-n}}$$

To find the number of periods required to pay back the loan at given amount per installment we can use:

$$n = \frac{lnp - \ln(p - rA)}{\ln(1 + r)}$$

Understanding Internal Rate of Return:

Just remember the following formula where Initial investment is A, and the returns per period is $p_1, p_2 \ldots\ldots\ldots p_n$ for n periods, The rate of return be r

$$A = \frac{p_1}{(1 + r)^1} + \frac{p_2}{(1 + r)^2} + \cdots \ldots + \frac{p_n}{(1 + r)^n}$$

To make the calculation easier assum$(1 + r)^{-1} = x$, therefore rewrite the formula